Verified Approaches to Inertial Navigation

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A new book by Professor Oleg S. Salychev, head of TeKnol Ltd., unfolds a long term experience of the author's leaded team in the field of inertial navigation system development. Facing actual challenges in design of INS line from low cost to medium accuracy units, the author introduces the readers into the specific of functioning and development of various navigation systems.

Vast scientific and research experience of TeKnol Ltd. is implemented in practical design the book describes in detail. It is intended for engineers and postgraduate students, who deal with the inertial instrumentation development.

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Contents

Introduction

Since the publication of the previous book, the spectrum of applications for the compact integrated inertial systems has come through a rapid extension. Genuine boom of commercially available UAVs is just a forerun to the coming era of mobile robotics. Inertial navigation system (INS) is a critical component to provide navigation and motion sensing, thus assuring an adequate control over the mobile system.

Since INS is a core component of the motion control system, its cost comes directly to the product price, hence, positioning the product on the market. With the relatively cheap MEMS and moderate-priced fiber-optic sensors that came in early 2000s, the use of INS was enlarged from aeronautical and military-grade products exclusively to wider, almost consumer market applications. However, anticipated revolution of MEMS sensors accuracy revolution is not yet here (and probably never won't). This is why the market pushes the developers to search for innovative cost-effective solutions that can satisfy increased accuracy requirement for mobile robots, UAVs, all kinds of autonomous vehicles.

The progress in digital processing, driven by Moore's law, enables on-board implementation of sophisticated algorithms, thus compensating the weakness of the hardware by the power of the software.

The initial part of this book provides the readers with the basics of inertial navigation, guides through the navigation algorithms, introduces into the estimation theory, capable of miraculous transformation of a trivial hardware pumpkin into the luxury carriage drawn by smart software horse power.

The source of magic is the task-oriented algorithm approach, referred as “multiplatform”, good for versatile applications: land, sea, air. The final part of the book is, in fact, a manual, providing the reader with knowledge essential to build software applicable for various life situations of integrated INS applications. Every approach considered was tried in actual practice during more than 20 years of author's experience with the field of applied inertial navigation.

Different approaches based on the external information may guarantee highly accurate output data. When external information is available, different error damping undisturbed by motion parameters can be applied. Much more problematic is the situation, when external navigation information is not available, as it requires different self-damping procedure. But, in principle, all of them are disturbed by nominal motion parameters such as accelerations. It seems reasonable to use different calculated platforms (implementations of the navigation frame) adjusted to the particular carrier motion. Each platform has its own damping control law and, as a result, particular different error frequency dominated behavior. The master filter combines different navigation platform solutions to create the optimal one for each mode of motion. Such approach is called the multiplatform navigation solution, and, with its help, the line of different low cost and medium accuracy INS have been developed over last 15 years.

The book unfolds the experience gained in the system design. It is dedicated those practically involved in the project: student, researches, engineers.

My sincere gratitude is to staff of TeKnol Ltd., namely D. Pazychev, V. Mkrtchyan for long time productive joint work and also to A. Levchenkov and A. Egorushkin for their help in book publication and effective project management. I also want to thank M. Ilina, U. Kostiuk and V. Voronov for inspiration.

Chapter 1

Coordinate Frames

In surveying and navigation positioning the final output required by a client usually includes the coordinates of a point, namely latitude, longitude and height and their accuracies. The measurements sensed by an Inertial Navigation System (INS) are three orthogonal components of the body rotation rates and three accelerations in a coordinate frame, having no direct link to any geodetic curvilinear coordinate frame. These measurements have to be analytically integrated and transformed through several coordinate frames, which yields changes in the ellipsoidal coordinates.

1.1 Inertial Frame

Newtonian definition says an inertial frame neither rotates nor accelerates. Easily defined theoretically, such a frame is almost impossible to implement in practice. The closest approach to a proper understanding of what a truly inertial frame is would be one that is inertial with respect to the distant stars. One approximation of such is a frame, used in surveying applications, which is a right ascension system. According to a catalogue, this system precesses and nutates at the rate lower than $3.6 \cdot 10^{-7}$ arc sec/s, well below noise level of inertial sensors employed by existing inertial

Chapter 2

Coordinate Transformation

2.1 Direction Cosine Matrix

Matrix for transformation an arbitrary vector from one coordinate frame to another can be presented as:

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_b = C_I^b \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}_I$$

where C_I^b is the transformation matrix between I frame and b frame.

The transformation matrix is often referred to as direction cosine matrix.

The most commonly employed method for coordinate transformation matrix generation is computation of the direction cosines between each pair of axes of two respective frames. One of the most commonly used methods consists in three consecutive right-handed rotations [2, 3, 5].

Angles of rotation, called Euler angles, define the position of one coordinate frame about another one.

Consider Euler angles $\mathbf{\Phi}_x$, $\mathbf{\Phi}_y$, $\mathbf{\Phi}_z$. Three consecutive right-handed rotations of these angles coincide two arbitrary frames.

Chapter 3

Principles of Inertial Navigation

3.1 General Navigation Equation

The core idea of inertial navigation is based on the acceleration integrations. A device for measuring vehicle acceleration is called an accelerometer. First integration of the vehicle acceleration produces velocity. Second integration gives vehicle position increments with respect to an initial point. To obtain navigation parameters (velocity, position, attitude) in a certain navigation frame, acceleration is to be projected on that frame. Different types of gyroscopes match sensitive axes of accelerometers with a certain navigation coordinate frame. Inertial system technique knows two approaches for the navigation frame simulations. First one deals with a physical implementation of the navigation frame using a three-axial gyro-stabilized platform with three orthogonally placed accelerometers. System of such type is called platform or gimbaled INS. The next approach, called strapdown INS (SINS), creates analytical image of the navigation frame in onboard computer by using measurements from accelerometers and rate gyros installed directly on a vehicle body.

To derivate the navigation equations for position and velocity, accelerometer measurements have to be introduced.

Chapter 4

Applied Navigation Algorithm

4.1 Strapdown System Navigation Algorithm

The total algorithm can be divided into two parts. The first part deals with the information processing of the accelerometer indications. The second one deals with the gyro output measurement preparation. Using the accelerometer biases estimates, scale factors and installation errors obtained by the factory calibration procedure (see Chapter 7), the compensation of the above errors is implemented to the raw accelerometer indications. After the above errors are compensated one can calculate velocity increments using the following formula

$$\Delta W_{x_b,y_b,z_b} = \int_{t_k}^{t_k+h_{N1}} a_{x_b,\mathrm{y}_b,z_b} dt \qquad (4.1)$$

where x_b, y_b, z_b — body frame; a_{x_b,y_b,z_b} — accelerometer output; h_{N1} — sampling period

The similar procedure is used for raw gyro output data. Here gyro biases, scale factors and installation errors are compensated as well. Angle increments can be calculated by the equation:

Chapter 5

INS Error Model

5.1 INS Error Model

Let us introduce small angles Φ_E, Φ_N, Φ_{up} between the platform and local-level frames. The above angles can be considered as an indicator of the system attitude errors. It is easy to explain using the fact that an ideal physical or calculated platform axes must coincide with the local-level frame or navigation frame. The transformation matrix between the local-level and platform frames can be described by the following formula (see equation (2.3)).

$$C_{LL}^{p} = \begin{bmatrix} 1 & \Phi_{up} & -\Phi_N \\ -\Phi_{up} & 1 & \Phi_E \\ \Phi_N & -\Phi_E & 1 \end{bmatrix} \tag{5.1}$$

Poisson equation for the direction cosine matrix between the navigation and platform frames has a form (see Chapter 2):

$$\dot{C}_{LL}^{p} = C_{LL}^{p}\breve{\omega}_{LL} - \breve{\omega}_{p}C_{LL}^{p}$$

where $\breve{\omega}_{LL}$ is a skew-symmetric matrix of the local-level frame absolute angular velocity; $\breve{\omega}_{p}$ is a skew-symmetric matrix of the platform frame absolute angular velocity.

Chapter 6

An Alternative Approach to the Inertial Navigation

6.1 Principle of Multiplatform Navigation

As it was mentioned in Section 5.5 traditional Schuler tuned solution is universal but not optimal for all possible motion modes of the aircraft.

For a relatively smooth flight different self-damped approaches, in comparison with traditional one, can provide results much more accurate. However, during high maneuvering self-damping causes severe disturbance of platform leveling. Assume that with the same raw data obtained from the sensors we can simultaneously create different platforms governed by different control laws. Each calculated platform implements its own navigation solution, which is based on certain control procedure. Moreover, each solution fits some motion mode and the global system output has to combine all solutions to create one uniform, acceptable for all possible motion modes. This approach can be called multiplatform navigation system [18]. Here each platform provides individual navigation solution and the master filter combines results obtained from analysis of vehicle motion mode. The idea is diagramed in Figure 6.1.

Chapter 7

INS Calibration and Testing

7.1 Factory Calibration of INS

In order to implement the navigation and alignment algorithms, the calibration of sensor errors has to be done previously (see Chapter 4).

Let's introduce the sensor error model.

$$
\begin{gathered}
\delta a_{bx} = \alpha_x + \alpha_{xx} a_{bx} + \alpha_{xy} a_{by} + \alpha_{xz} a_{bz} \\
\delta a_{by} = \alpha_y + \alpha_{yx} a_{bx} + \alpha_{yy} a_{by} + \alpha_{yz} a_{bz} \\
\delta a_{bz} = \alpha_z + \alpha_{zx} a_{bx} + \alpha_{zy} a_{by} + \alpha_{zz} a_{bz} \\
\delta\omega_{bx} = \beta_x + \beta_{xx}\omega_{bx} + \beta_{xy}\omega_{by} + \beta_{xz}\omega_{bz} + \\
+(\beta_{xyx} a_{bx} + \beta_{xyy} a_{by} + \beta_{xyz} a_{bz})\omega_{by} + (\beta_{xzx} a_{bx} + \beta_{xzy} a_{by} + \beta_{xzz} a_{bz})\omega_{bz} \\
\delta\omega_{by} = \beta_y + \beta_{yx}\omega_{bx} + \beta_{yy}\omega_{by} + \beta_{yz}\omega_{bz} + \\
+(\beta_{yxx} a_{bx} + \beta_{yxy} a_{by} + \beta_{yxz} a_{bz})\omega_{bx} + (\beta_{yzx} a_{bx} + \beta_{yzy} a_{by} + \beta_{yzz} a_{bz})\omega_{bz} \\
\delta\omega_{bz} = \beta_z + \beta_{zx}\omega_{bx} + \beta_{zy}\omega_{by} + \beta_{zz}\omega_{bz} + \\
+(\beta_{zxx} a_{bx} + \beta_{zxy} a_{by} + \beta_{zxz} a_{bz})\omega_{bx} + (\beta_{zyx} a_{bx} + \beta_{zyy} a_{by} + \beta_{zyz} a_{bz})\omega_{by}
\end{gathered}
\tag{7.1}
$$

Chapter 8

Introduction to an Applied Estimation Theory

8.1 State Space Representation of Linear Systems

The state space representation converts a n^{th} order system of differential equations into n coupled first order differential equations. Such representation is often desirable in many applications, because it allows the usage of vector-matrix techniques [9, 10, 11, 14]. In the state space form any linear system can be described as

$$\dot{x}(t) = A(t)x(t) + B(t)w(t) \tag{8.1}$$

where $x(t)$ is $(n\times 1)$ state vector; $A(t)$ is $(n\times n)$ system matrix; $w(t)$ is $(r\times 1)$ input (system) noise, assumed to be white with zero mean; $B(t)$ is $(n\times r)$ input matrix.

The part of state vector components or its linear combinations can be physically measured according to the model

$$z(t) = H(t)x(t) + v(t) \tag{8.2}$$

where $z(t)$ is $(m\times 1)$ measurement vector; $H(t)$ is $(m\times n)$ measurement matrix; $v(t)$ is $(m\times 1)$ measurement noise, assumed to be white with zero mean.

Chapter 9

INS/GNSS Integration

9.1 INS/GNSS Integration Methods

Table 9.1 compares INS and GNSS properties.

Table 9.1

INS and GNSS properties comparison

Properties	INS	GNSS
Output rate	High (100...200 Hz)	Low (1...50 Hz)
Long term accuracy	Low	High
Short term accuracy	High	High
Independence	Self-contained unit	Dependent on direct availability of satellites
Possibility of attitude measurements	Compact and reliable	Dependent on environment

The comparison clearly demonstrates that a GNSS receiver and an INS have complimentary characteristics. An INS is a self-contained unit, which provides information on position, velocity and attitude regardless of external environment. Moreover, the INS has a high output rate limited mostly by the computational approach and equipment. High-frequency noise of the inertial instruments is attenuated by the INS low-pass nature. As a result, the INS error

Chapter 10

Land Application of INS

10.1 Inertial Navigation System Integrated with Odometer

An odometer is a device to calculate the position increments of along the longitudinal axis of the vehicle. It means that the coordinate increments in local-level frame for North-East directions can be rearranged through the odometer measurements as Figure 10.1 shows.

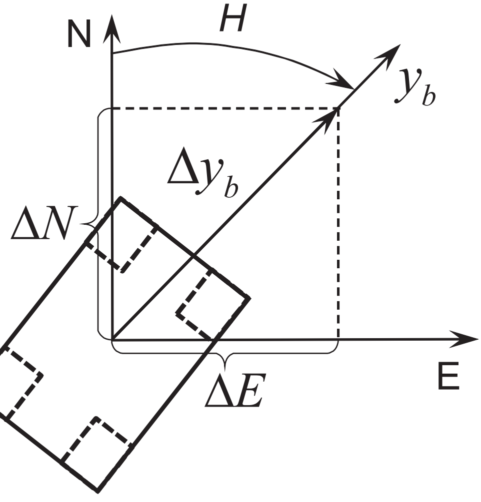

Figure 10.1 Vehicle body position

Chapter 11

Airborne Application of INS

11.1 Application of the Multiplatform Approach to the Low Cost INS

11.1.1 Implementation of the Traditional Approach to the Low Cost INS

The low cost MEMS sensors (accelerometers and rate gyros) are now being manufactured by many companies around the world: Analog Devices, Silicon Sensing Systems, Systron Donner etc. [3]. Typical performance of such gyros and accelerometers is shown in Table 11.1.

Table 11.1

Performance	**Gyro channels**	**Accelerometer channels**
Range	150 — 300 deg/sec	5g — 10g
Run to run bias	1 deg/sec	10mg
In run bias stability	0.1 — 0.01 deg/sec	3 — 5mg
Resolution	8 — 15 deg/h	$1{\cdot}10^{-4}$ g

Appendix A

Calibration of Accelerometer Scale Factor

Note that nonstationary components are weakly observable when external information is used in real time. The reasonable way for their removing is a special preliminary calibration procedure.

The non-stationary component of the INS velocity errors is described as (see Chapter 3):

$$\delta V_E^{nst} \approx \Phi_{up} V_N + \mu_E V_E$$

$$\delta V_N^{nst} \approx -\Phi_{up} V_E + \mu_N V_N \tag{A.1}$$

where μ_E, μ_N — accelerometer scale factors.

Note that Φ_{up} is different for each system turn and has to be calculated for each run. Accelerometer errors μ_E, μ_N can be assumed constant at least for some time of INS exploitation. It means they can be estimated and compensated with a special calibration procedure. One of the best approaches for their calibration is to use a special vehicle motion. Assume, that INS is installed on a land vehicle, which moves along a trajectory shown in Figure A.1.

Here, at the first stage of the system calibration, an IMU is installed so that y_b axis coincides with the longitudinal axis of the vehicle. The vehicle periodically stops at some points and its true velocity equals to zero. Hence the INS's velocity indications (called zero velocity updates (ZUPT)) are velocity errors. 2.5...3 min is a

Appendix B

Scalar Adaptive Estimation

This part considers a scalar approach to state vector estimation that differs from the known procedures by its capability to form an estimation equation independently for each observable component of the state vector [12].

Scalar algorithms are low-sensitive to an accuracy of the mathematical model description and input noise statistics due to an adaptive adjustment of the gain coefficient depending on the current estimation errors. The implementation of the algorithms leads to the application of an analytic expression of the formula independently for each observable state vector component.

Independent Scalar Estimation of the State Vector Observable Component

Assume the object is described by the following equation:

$$x_k = \Phi x_{k-1} + G w_{k-1} \tag{B.1}$$

where x_k — ($n \times 1$) state vector; Φ — ($n \times n$) transition matrix; G — ($n \times r$) input matrix; w_k — ($r \times 1$) input noise vector assumed to be a white Gaussian noise with zero mean.

The measurement equation has a form

$$z_k = H x_k + v_k \tag{B.2}$$

where H — ($1 \times n$) measurement matrix, that, to simplify the further calculation is assumed to be a line matrix (not affecting the

Appendix C

Application of an Adaptive Filter for the Acceleration Smoothing

Consider an example of adaptive filter implementation for estimation of acceleration errors using current local-level accelerations as the measurements [18].

In this case the measurement model has the following form:

$$z(E) = \delta\dot{V}_E + a_E(true)$$

$$z(N) = \delta\dot{V}_N + a_N(true)$$

here $\delta\dot{V}_E$, $\delta\dot{V}_N$ are acceleration errors, $a_E(true)$, $a_N(true)$ are true values of the local-level accelerations, which represent the measurement noise.

System model for the Kalman filter is based on the INS error model.

The most simple model for one channel (for instance, E) has a form:

$$\delta\ddot{V}_E + \nu^2\delta V_E = -g\omega_N^{dr} - g\omega_N^{c}$$

$$\dot{\omega}_N^{dr} = -\beta\omega_N^{dr} + A\sqrt{2\beta}w_1(t) \tag{C.1}$$

here ω_N^c is damping signal, which optionally (see Chapter 6) may have a form

Appendix D

Wave Estimation of the State Vector

Wave approach to the estimation problem has been suggested in [12].

D.1 System Model Representation

As it was mentioned in Chapter 8, the accuracy of estimation algorithm is restricted by the level of uncertainties in the system model representation. The model creation is a key for the eligible estimation algorithm design. Obviously, the model representation has to depend on the knowledge on the real processes taking place in the system as well as on the time scale of their description. The philosophical interpretation of the system model creation is shown in Figure D.1.

Here God, who covered all points in infinite space, has no time, and, as a result, absolute information. Using deterministic model he can predict or restore information with absolute accuracy.

The human has a restricted time scale, which leads to the appearance of some uncertainties, which could not be described in deterministic bounds only. In this case the problem is to reduce the influence of random part of the system description. Obviously,

Appendix E

GNSS principle

The pseudorange measurements can be made using only a code, because only the use of a particular code can give the indication of a time mark sent out from the GNSS satellite and detected by the receiver firmware [4, 5].

Knowing the transmit time of event (a) when a signal is sent out and the received time of event (b) when a signal is received, one can compute a pseudorange

$$\tilde{\rho}_r^i = c[T(b) - t^i(a)]$$

where $\tilde{\rho}_r^i$ — pseudorange between a receiver and i-satellite;

$T(b)$ — received time when a signal is received; $t^i(a)$ — transmit time when a signal is sent; c — speed of light.

Relating the two different time scales with the reference GNSS time, one gets

$$\tilde{\rho}_r^i = \rho_r^i + c\Delta tr$$

where ρ_r^i — geometric distance between the receiver and i-satellite; Δtr — receiver clock offset from the satellite system time.

An arbitrary pseudorange can be rearranged through the coordinates of i-satellite and coordinates of the receiver as

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